RRN61

RADIATION FROM INTENSE BEAMS IN LARGE MAGNETIC FIELDS

by

David C. dePackh

There is under consideration a possibility for generating high levels of interesting radiation through the injection of kiloampere beams of GeV electrons into megagauss magnetic fields. This note is intended to examine certain features of this notion.

Jackson[1] treats the question of so-called synchrotron radiation from relativistic electrons for single particles and without regard to their containment and slow retardation. The important features of this radiation as they bear on the present problem are:

(1) Spectral shape. The radiation from a single electron in a uniform magnetic field H is characterized by the formula for the energy per unit angular frequency per electron revolution

$$I(\omega) = \sqrt{3}\,(e^2/c)\,\gamma\,2u\int_{2u}^{\infty} K_{5/3}(v)\,dv \equiv (e^2/c)\,\gamma\,f(u)$$

where $u \equiv \omega/\omega_c$ and $\omega_c \equiv 3\gamma^2\,cH/1700$. The latter quantity is the "critical frequency", beyond which the spectrum is exponentially falling. A plot of the spectrum appears in figure 1; it is seen that the peak occurs at about 0.2 of the critical frequency. The photon energy at the critical energy is

$$E_c = 3.47 \times 10^{-8} \gamma^2 H \text{ ev.}$$

For $\gamma = 2000$ ($\sim$ 1 GeV) and $H = 10^6$ gauss, one has for E_c the value 139 keV (spectral peak about 28 keV).

(2) Rate of energy loss. From the formula in ref. 1 one can derive the result

$$d\gamma/dt = -1.88 \times 10^{-9} H^2\gamma^2.$$

If this is divided by γ/t_r, where t_r is the orbital period, the result is

$$t_r d\gamma/\gamma dt = -6.69 \times 10^{-16} H\gamma^2.$$

giving the relative energy loss per turn. For the above parameters this quantity is about 0.0027, meaning that about 370 turns are needed to produce an appreciable energy loss (that is, about 260 nsec). Thus the electrons must be stored in the field for at least this long for effective delivery of radiation.

From these formulae one can derive the spectrum resulting from the coming to rest of an electron in the magnetic field.

Since $I(\omega)$ is the energy per revolution, the energy per second is

$$P(\omega) = Ic/2\pi r = (e^2H/1700\ \pi)\ f(u).$$

As written, this formula holds for a single electron. If there are $N(\gamma)d\gamma$ electrons in $d\gamma$ at γ, it becomes

$$P(\omega) = \int_0^{\gamma_o} (e^2H/1700\ \pi)\ N(\gamma)\ f(u)\ d\gamma.$$

If N_o electrons are injected per second, then $N(\gamma)d\gamma = N_o d\gamma/(d\gamma/dt)$, and so

$$P(\omega) = (e^2N_o/3.2 \times 10^{-8}\ \pi H) \int_0^{\gamma_o} f(u)\ d\gamma/\gamma^2$$

$$= 2.13 \times 10^{-14}\ (N_o/\gamma_o\ H) \left[u_o^{-\frac{1}{2}} \int_{u_o}^{\infty} u^{-\frac{1}{2}} f(u) du\right].$$

The quantity in brackets, called $g(u_o)$, is plotted in figure 1 as a function of $u_o \equiv \omega/\omega_{co}$, where ω_{co} is the critical frequency at the injection energy γ_o. The soft end of the spectrum can be shaped by the use of absorbers. The high efficiency (~ 100%) and rather sharp cutoff of the spectrum are attractive for many purposes, as is the fact that the beam involves ν/γ values far below unity, making for truly beam-like flow.

The obvious problems are connected with (1) trapping of the beam in the field, (2) generating the beam, and (3) generating the magnetic field. Other difficulties may arise from the fact that a rather long time is required for delivery of the radiation. Only the three questions enumerated will receive any consideration here, and these only in a speculative manner.

Trapping

An obvious time of interest is the number of turns a hypothetical nonradiating electron can remain in the field before being ejected by Liouville's theorem. This time depends, among other things, on the departure of the electron from the design optimum and so is affected by the quality of the injected beam. The optimization of the magnetic field configuration so as to maximize the containment time is the principal problem of the magnetic field design. Because the radiation loss is relatively small ($t_r d\gamma/\gamma dt$ is considerably below unity), it seems probable that the magnetic field will have to increase in time during injection (figure 2). For hypothetical numbers, suppose the nonradiating electron can remain in the field 10 turns before leaving. In this time it might have lost about 3% of its energy by radiation. Suppose a 10% energy loss is sufficient to insure that the particle is trapped in the field; then the field must increase 7% during these 10 turns to insure capture. For a beam with an initial orbital period of 1 nsec and a total duration of 100 nsec this means an overall increase of the order of 100% during injection. This could be reduced by improvements in the field design, and these figures are not intended to be in any way typical, but only to show the kind of consideration which must go into the design of the trapping field. It should be clear that such problems are important and nontrivial in this system, the reason being that the loss by radiation, even with the presently considered parameters, is a fairly slow process, so that the field must be made to accommodate the electrons for a rather long time.

Beam Production

Consider a beam of 1 ampere at 1 Gev; this energy is chosen because it makes a good fit to interesting radiation spectra at magnetic fields which should be within reach of present technology. The beam power is 10^9 watts; the beam energy at 100 nsec pulse length is 100 joules, most of which is converted into radiation with the spectrum previously described or something close to it. A comparable electron beam at 500 kev producing bremsstrahlung by conversion in a material target at reasonable efficiency (~0.5%) would require about 20 kJ of electrons, or 400 kA. Systems of the order of 10 times this current are presently under consideration, thus giving possibly 1 kJ of radiation; this would correspond to about 10 amperes in a 1-Gev beam. Comparing the two methods of producing radiation at <u>this</u> level of intensity, one has little difficulty in choosing the low-energy, high-current bremsstrahlung system, despite the low efficiency, because of the relative ease of the technology (setting aside as soluble all questions connected with large ν/γ). From this point of view there is therefore little advantage in considering Gev systems with currents less than, say, 100 A. One then looks at the two natural sources for intense, energetic beams: linear accelerators and circular devices, which latter class may be divided into accumulation and single-turn injection systems.

Linear accelerators for this application could be of rf or induction type. In all linear systems the instantaneous power requirements are very great for really interesting beams (~1 kA), since the beam power is 10^{12} watts. For a device of practical length the demands in a supply scheme in which power is delivered to the beam only while it is actually passing through are almost certainly beyond present technology (order of several Gw per foot), and therefore one looks for systems in which the required energy is stored in the device prior to beam passage. A rough idea of the requirements for a stored-energy linear accelerator can be obtained by first setting a breakdown limit on energy gain per cm per electron, say $E_o e$, so that the total energy gain per cm is $iE_o t_o/c$, where t_o is the pulse length. Next define a factor k related to the permissible energy degradation during the pulse, so that the stored rf energy is $kiE_o t_o/c$. Now define a geometrical factor h, so that the rf stored energy per cm is $h(E_o^2/8\pi)(\pi a^2)$, where a is the cavity radius; for a cylinder in the TM_{010} mode h~0.29. Then on equating the required stored energy (so that the beam energy degradation will be within the prescribed limits) with the energy stored in the cavity, one has in practical units (amperes, volts/cm, seconds)

$$a = 2.6 \times 10^6 \sqrt{(k/h)(i/E_o)t_o}.$$

In a TM_{010} cylindrical cavity one has approximately $\lambda = 2.6$ a for the wavelength. Therefore if i = 1000 amperes, $E_o = 3 \times 10^5$ v/cm, $t_o = 10^{-7}$ sec, k/h = 10, then

$$a \sim 150 \text{ cm}, \lambda \sim 390 \text{ cm, frequency} \sim 77 \text{ Mhz}.$$

For the postulated TM_{010} cavity with length L much less than the diamater the Q-factor is

$$Q \sim L/d,$$

where d is the skin depth. By definition of Q, one has also

$$Q = (\text{energy stored/power dissipated})\omega;$$

because of the geometry energy and power can be taken per unit length. Equating the two expressions for Q, we have

$$P = (d/L)\ kiE_o t_o$$

for the power required to supply cavity losses at the indicated level of stored energy. If k=3, $ikE_o t_o$ = 90 joules/cm, d = 0.6 x 10^{-3} cm for copper at 77 Mhz, then

$$P = 27/L \text{ megawatts/cm},$$

that is, the power requirement per cavity is 27 megawatts, approximately independent of the length, as long as this remains materially below the diameter. It is thus of obvious benefit to use long cavities; this tendency may, however, be tempered by large static voltages existing in long cavities. The time required to charge the cavity is Q/ω, where Q may be of the order of 10^5, while ω at 77 Mhz is about 5 x 10^8. Engineering problems may be encountered in supplying the required power level for such long pulses. It is interesting to note that for sufficiently large currents the pulse length approaches $1/\omega$, and the device then becomes critically damped; it is then appropriate to characterize it as an air-core induction accelerator. For such extremely intense beams, and even for the presently considered beam, the air-core device is superior to the iron-core induction accelerator, which is limited by nonlinearity, breakdown, and core losses.

The beam can acquire its energy more slowly in long-containment devices, where the beam is captured in a circular orbit, although such long containment increases the danger of instability development. Kiloampere currents might be obtained either by injection in a single turn (Sozotron-type device) or by accumulation (Budker-Naumov accelerator[2], FFAG device[3], or storage ring). The Sozotron[4] is a high-phase-space-density device, requiring very strong focusing or high injection energy to maintain the beam diameter. Increase of the injection energy to 10 Mev would probably permit sufficient reduction in the strength of the focusing field compared to that of the present Sozotron design to allow extension of its energy to 1 Gev with practical energy storage in the focusing field. This also allows injection above the critical energy. The Novosibirsk B-3 machine was intended to produce kiloampere beams by accumulation in radius, but it is presently doubtful whether this machine has fulfilled its design objectives. Budker's group claims to have reached a space-charge-limited current of about 75 amperes in the B-2 machine[2] by radial accumulation in a large, flat aperture, this being the highest current reported anywhere in a circular vacuum machine. The MURA FFAG machine raised a number of 0.1-ampere injected beams to a stacking energy of 35 Mev, where a current of about 6 amperes was reached before instability onset. Beams produced by accumulation are of inherently low phase-space density and for that reason may not be ideally suited for the present application. However, momentum spreads may be helpful in the suppression of instability, and if such beams are stored at moderately high energy, their phase-space spread may be reduced by radiation.

The problem of stability is not well understood. All high-current circulating beams except that of the Novosibirsk B-2 have reported transverse and/or longitudinal instability. Transverse instabilities, of rather long growth time, have been corrected by inverse feedback and by nonlinearity. The situation with regard to longitudinal instability is less satisfactory. Longitudinal instability with growth times much shorter than have been predicted by a linear resistive-wall theory have been observed in the Sozotron and in the MURA machine. A nonlinear theory based on perfectly conducting walls is under consideration by the Sozotron group; preliminary computer investigation has confirmed the possible existence of such effects and has shown that substantial amounts of the beam kinetic energy can be deposited in the potential energy of short longitudinal bunches. These instabilities may also be controllable by inverse feedback; their rapid growth and short wavelength are likely, however, to impose severe engineering problems. Computer analysis of rf-bunched beams has not yet been undertaken, but only minor changes in present programs are needed to provide for such beams. The present Mark-I Sozotron can also be used for studying these instabilities experimentally.

At the present stage of the consideration of these problems it is certainly premature to make meaningful comparisons among the various accelerating systems. However, as the situation can be discerned from the above remarks, one is inclined to favor the linear accelerator for several reasons: (1) the short containment time and rapid rate of energy gain are desirable for the suppression of instability; (2) the design is comparatively straightforward, so that a working device should be obtainable in minimum time and with a minimum of beam physics research; (3) the phase-space density should ideally be high, making the accelerator a good injector for the radiation generator; and (4) the beam extraction problem is nonexistent. A minor advantage is the absence of radiation loss when it is not wanted.

Generation of the Intense Magnetic Field

In other phases of DASA-supported work there are numerous high-energy fast pulse lines either under development or already in existence, and currents well in excess of 1 Ma are being considered. It is not presently possible to describe in any detail the sort of geometry in which the field should exist. If it is supposed that the field should occupy a volume of about 5 cm^3 and should be of the order of 1 Mg throughout this volume, the resulting magnetic energy is then about 20 kJ, which is by no means an impossible level. The use of a pulsed line means that the field can be turned on nonadiabatically, which is almost certainly necessary for a circular beam source (circular accelerator or storage ring) and is likely also to be necessary for a linear accelerator source in view of the need for a time-increasing field for trapping the orbits. Ideally the field is better supplied by purely electrical means than through the use of high explosives. The field for 1-Ma currents in 1-cm geometries is of the order of $0.4\pi I/w \sim 1.3$ Mg, so that the orders of magnitude are reasonable. The source should allow of a containment time of the order of 1 microsecond at most.

In the circular device one imagines a slotted structure into which the beam can be adiabatically shifted prior to the turn-on of the trapping field (figure 3). Material of which this structure is made should be transparent to as much of the radiation as possible both for the sake of the spectrum and to minimize the danger of destroying the structure; beryllium seems a natural choice. The sketch in figure 3 is obviously only schematic, since much attention will have to be given to problems of field shaping so as to optimize the trapping. If a linear accelerator is used, some simplification should be possible since transverse adiabatic shifting prior to field turn-on is no longer needed. In any of these systems the beam should be of as small diameter as possible both to minimize the magnetic energy of the field and to improve the trapping.

Conclusion

It cannot presently be said that the proposed way of generating intense electromagnetic radiation is practicable. The rewards for a successful device along these lines are sufficiently great, however, that a feasibility study should be made of the following points: (1) is it possible to design a megagauss magnetic field having a configuration such as to insure trapping of orbits produced by a practical electron source? (2) what kind of electron source can best meet the requirements for injection into such a field, having regard for the fact that the system may have to meet some sort of time schedule? (3) what are likely to be the major causes of instability in any of these sources, and how can they be overcome? and (4) how can the currents needed for the intense magnetic field be produced? It seems probable that the expenditure of approximately two man-years of effort should provide enough answers to make possible a decision on further pursuit of this approach.

References

1. J. D. Jackson, Classical Electrodynamics, Wiley, 1962, pp 481-488.

2. E. A. Abramyan et al, Soviet Physics-Tech. Phys. 10, 477 (1965).

3. MURA Staff, Int. Conf. on High-Energy Accel., CERN, 1959, p. 71.

4. D. C. dePackh, Report of NRL Progress, March 1966, p. 9.

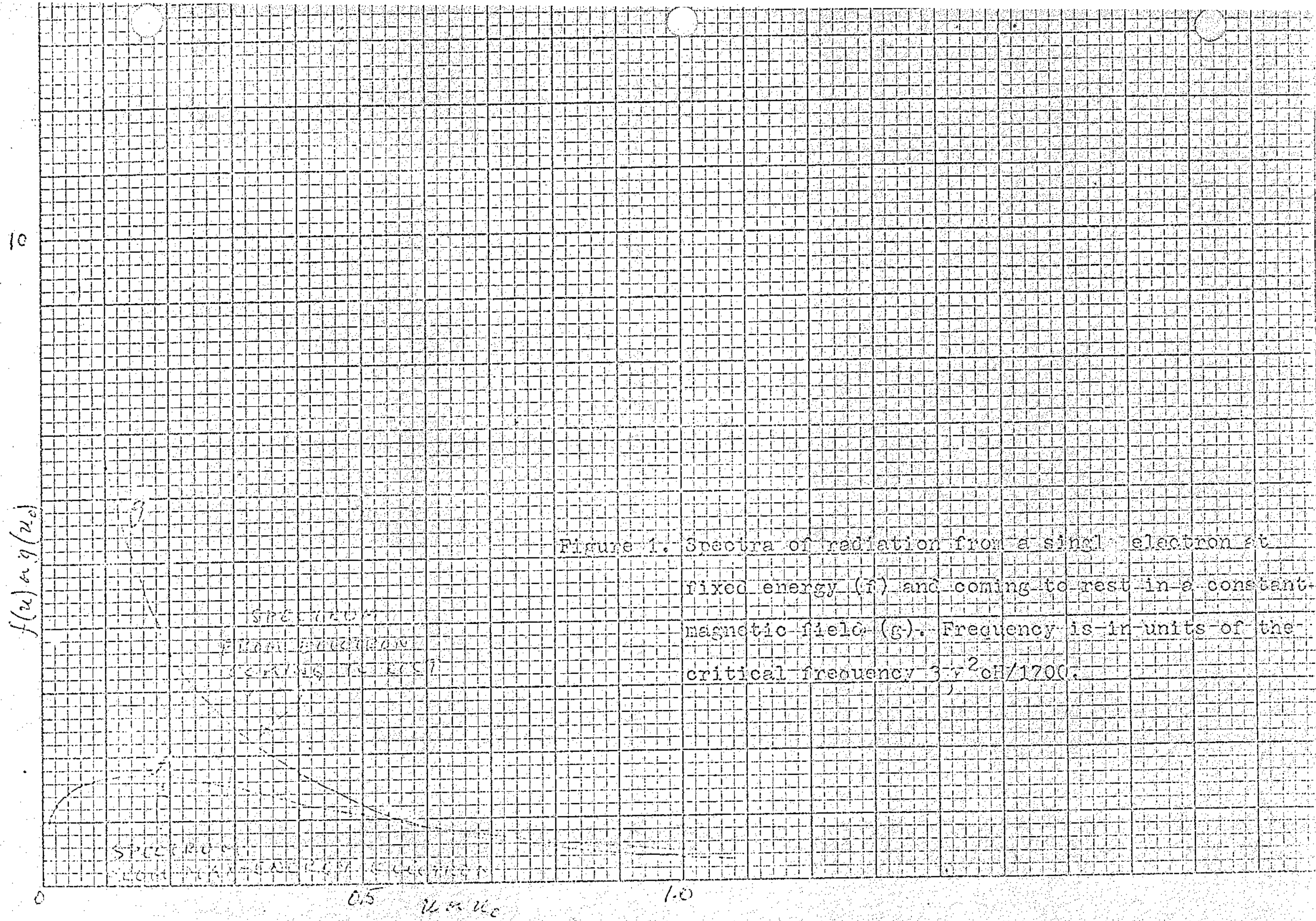

Figure 1. Spectra of radiation from a singl electron at fixed energy (f) and coming to rest in a constant magnetic field (g). Frequency is in units of the critical frequency $3\gamma^2 cH/1700$.

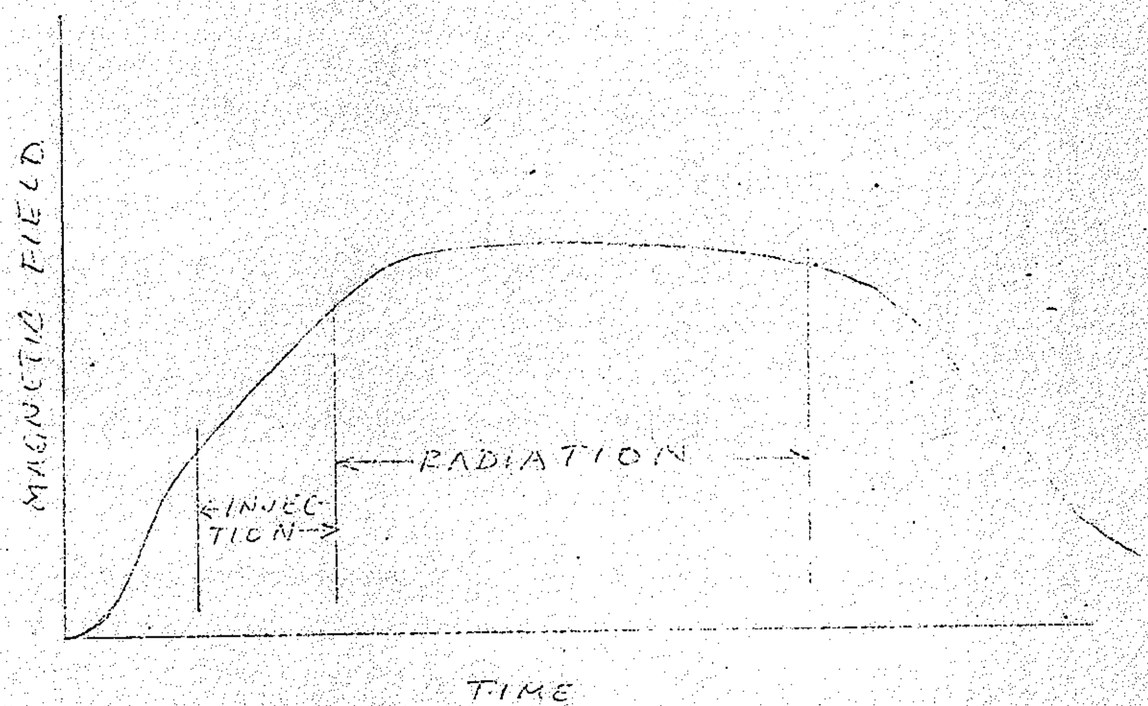

Figure 2. Schematic of magnetic-field program for generation of synchrotron radiation

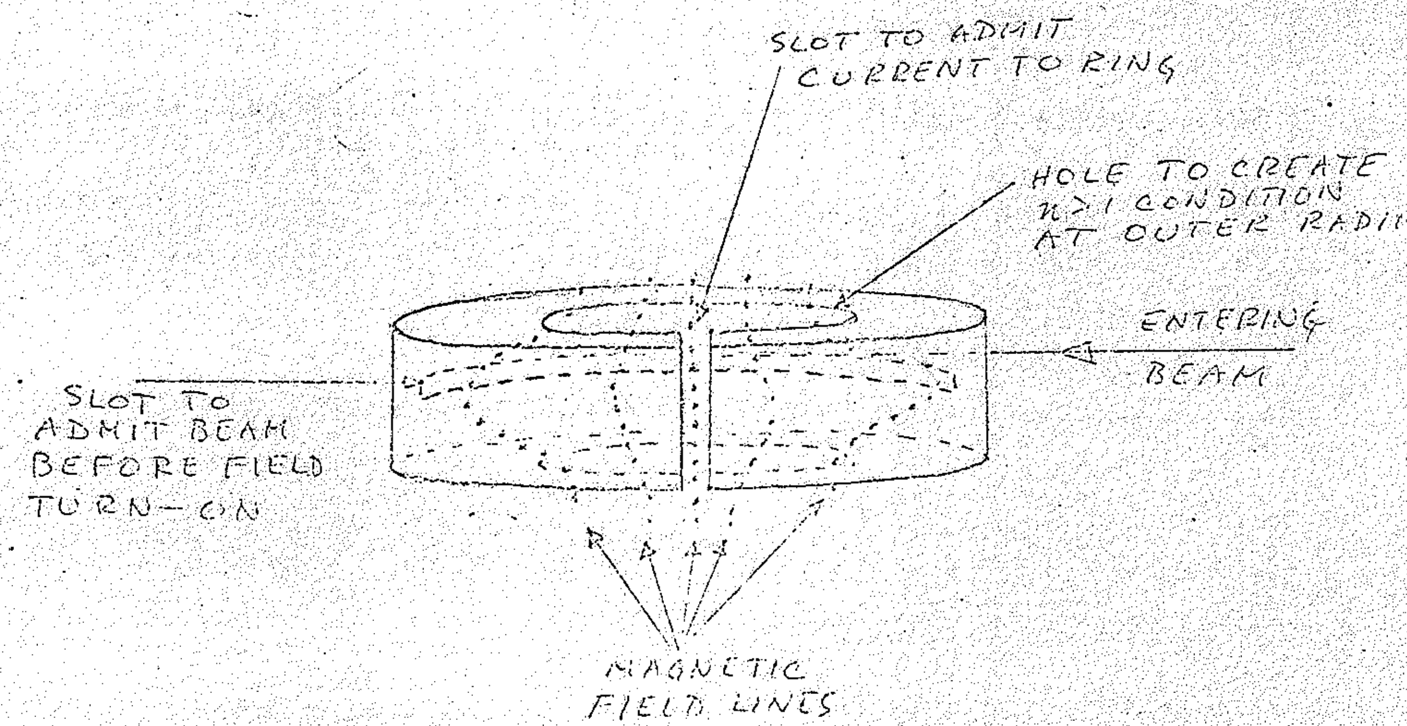

Figure 3. Schematic of current loop for formation of short megagauss field pulse